MATHEMATICS

Syllabus for Primary Schools

RATIONALE

THE MATHEMATICS PROGRAMME

For Primary Schools

Rationale

Mathematics is a logical, reliable and growing body of concepts, which makes use of specific language and skills to model, analyse and interpret the world. It provides a means of communication that is powerful, concise and precise.

As a human activity mathematics involves creativity in the discovery of patterns of shape and number, the recognition of relationships, the modelling of situations, the interpretation of data and the communication of emerging ideas and concepts.

Mathematics is one of the essential areas of learning. Everyone needs to develop mathematical concepts and skills to help them understand and play a responsible role in society. Mathematics education aims to provide students with those skills and understandings.

The need for people to be numerate has always been identified as an important outcome. Mathematics education aims to contribute to the development of a broad range of numeracy skills.

In an increasingly technological age the possession of problem solving and decision making skills is an essential requisite. Mathematics education provides the opportunity for students to develop these skills and encourages them to become flexible problem solvers.

The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have been highlighted in the NMC as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem solving situations, thereby contributing to the development of various social and cooperative skills.

Increasingly, information is communicated through the use of graphical displays. The communication of information in graphic form is particularly common in mass media. It is important that people are able to draw sensible conclusions from charts, tables and graphs. At the same time increasing numbers of jobs demand the ability to collect data, to understand and use information technology for the organisation and interpretation of data, and to present reports and summaries. Mathematics education gives young people the opportunity to develop information skills through learning and practising data handling and data interpretation.

General Aims of Mathematics Education

Mathematics education aims to:

- help children appreciate the value of mathematics and its usefulness to them, to develop confidence in their own mathematical ability, to foster a sense of personal achievement and to encourage a continuing and creative interest in mathematics;
- develop in children the skills, concepts, understandings and attitudes which will enable them to cope confidently with the mathematics of everyday life;
- help children develop a variety of problem solving strategies involving mathematics and develop the ability to think and reason logically;
- help children become mathematically literate to in a world which is technologically oriented and information rich;
- provide a foundation for those children who may wish to further their studies in mathematics or other subjects where mathematical concepts are essential.

Achievement Aims of the Mathematics Curriculum

Number and Algebra

The mathematics curriculum provides opportunities for children to:

- develop an understanding of numbers, the ways they are represented and the quantities for which they stand;
- develop accuracy, efficiency and confidence in calculating – mentally, and on paper;
- develop an ability to estimate and to make approximations, and check the reasonableness of results and measurements;
- recognise patterns and relationships in mathematics and the real world;
- develop the ability to use symbols, notation, graphs and diagrams to represent and communicate mathematical relationships and concepts.

Measures, Shape and Space

The mathematics curriculum provides opportunities for children to:

- gain a knowledge of geometrical relations in two and three dimensions and recognise and appreciate their occurrence in the environment;
- develop spatial awareness and the ability to recognise and make use of the geometrical properties and symmetries of everyday objects;
- develop the ability to use geometrical models as aids to solving practical problems in time and space;
- develop knowledge and understanding of systems of measurement and their use and interpretation;
- develop confidence and competence in using instruments and measuring devices.

Data Handling

The mathematics curriculum provides opportunities for children to:

- recognise appropriate statistical data for collection and develop the skills of collecting, organising and analysing data;
- interpret data presented in tables, charts and graphs of various kinds.

Problem Solving

The mathematics curriculum provides opportunities for children to:

- develop flexibility and creativity in applying mathematical ideas and techniques to unfamiliar problems arising in everyday life and develop the ability to reflect critically on the methods they have chosen;
- become effective participants in problem solving teams, learn to express ideas and listen and respond to the ideas of others;
- develop the skills of presentation and critical appraisal of a mathematical argument or calculation, use mathematics to explore and conjecture and learn from mistakes as well as successes;
- develop the characteristics of logical and systematic thinking and apply these in mathematical and other contexts, including other subjects of the curriculum;
- develop the skills and confidence to use the language of mathematics to express mathematical ideas.

Approaches to Teaching and Learning in Mathematics

Problem Solving Approach

A balanced mathematical programme incorporates concept learning and the development, maintenance and application of skills. These should be taught in such a way that children develop their ability to think mathematically.

Children learn mathematical thinking most effectively through the application of concepts and skills in interesting and realistic contexts that are personally meaningful to them. This implies that mathematics is best taught by helping children to solve problems drawn from their own experiences.

Real-life problems are not always closed, nor do they necessarily have only one solution. Determining the best approach for solving a problem when several approaches are possible is a skill frequently required in the workplace. Consequently children need to be given various opportunities to work on open-ended problems. The solution to problems, which are worth solving, rarely involves only one item of mathematical understanding or just one skill. Rather than remembering a single correct method, problem solving requires children to search for clues and make connections to the various pieces of mathematics and other knowledge and skills, which they have learned. Such problems encourage thinking rather than mere recall.

Closed problems, which follow a well-known pattern of solution, develop only a limited range of skills. They encourage memorisation of routine methods rather than experimentation and investigation. Without diminishing the importance of being fluent with basic techniques, routine methods only become useful tools when children can successfully apply them to non-routine and realistic problems.

Good problem solving techniques are characterised by the systematic collection of data or evidence, experimentation (including trial and error followed by improvement), creativity, reflection on and critical evaluation of the process that has been followed. These characteristics may be developed by providing children with opportunities that encourage them to practise and learn simple strategies such as guessing and checking, drawing
a diagram, making lists, looking for patterns, classifying, substituting, re-arranging, putting observations into words, making predictions and developing simple proofs.

Learning to communicate about mathematics and through mathematics is part of learning to become a mathematical problem solver and learning to think mathematically. Critical reflection may be developed by encouraging children to share ideas, to use their own words to explain their ideas and to record their thinking in a variety of ways, such as words, symbols, diagrams and models.

The Language of Communication

"*The National Minimum Curriculum encourages at this* (Primary) *level to use English when teaching . . . Mathematics . . . In classroom situations when teaching these subjects in English poses difficulties, code switching can be used as a means of communication."* (NMC p. 79) The NMC document makes a similar recommendation regarding the use of English as the language of communication and the use of code switching where English "*poses great pedagogical problems*". (NMC p. 82)

In view of these recommendations it is up to the class teacher to decide what language must be used to facilitate the development and acquisition of mathematical concepts. Once this objective is achieved, however, it is essential that children be exposed to the mathematical ideas in English and listen to adults using the words correctly. Care must be taken to ensure that the English language used is simple and accessible; hence it should be presented in very short sentences in situations involving the appropriate mathematical language. The use of flash cards, displaying the specific mathematical language, is recommended. In the younger classes pictures and real-life objects should be used to facilitate the children's understanding of the language, as do consistency and repetition. As they grow older children should be encouraged to express and articulate their explanations, thinking and reasoning in English to strengthen their mathematical communication skills. However, on no account should the use of either language (Maltese or English) be to the detriment of children learning mathematics.

Developing the Understanding of Mathematical Vocabulary

Children's failure to understand mathematical vocabulary manifests itself when they fail to answer questions during lessons, when they fail to carry out a set task and when they do poorly in tests and examinations.

Possible reasons for this failure could be that:

- **they do not understand the spoken or written instructions;**
  (e.g. *draw a line . . .; put a ring around one of these numbers . . .*)
- **they are not familiar with the mathematical vocabulary;**
  (e.g. *difference, product, prime, multiple, factor, estimate . . .*)
- **they may be confused about mathematical terms which have different meanings in English;**
  (e.g. *table, volume, odd, . . .*)
- **they may be confused about other words.**
  (e.g. *sides* and *size*; *width* and *with*; *height* and the Maltese *ħajt*; *straight, vertical and horizontal . . .*)

It is for these reasons that children need to acquire the appropriate mathematical vocabulary so that they can fully participate in set tasks and tests. An
even more important reason is that mathematical language is crucial to the children's development of thinking. Unless they have the vocabulary to talk about *division*, *perimeter*, *capacity*, *etc*, they cannot make progress in understanding the various areas of mathematical knowledge.

Since children cannot learn the meaning of words in isolation, the use of questions is crucial in coming to grips with the mathematical ideas and mathematical terms correctly. It is important to ask questions in different ways so those children who do not understand the first time may subsequently pick up the meaning. One should not use only questions that require recall and application of facts but also questions, which require a higher level of thinking and promote good dialogue and interaction. Eventually children will begin to give more complex answers in which they explain their thinking.

All children need regular, planned opportunities to develop their mathematical vocabulary. They need to experience a cycle of **oral work**, **reading** and **writing**.

They need **oral work based on practical work** so that they may have visual images and tactile experience of what mathematical words mean in a variety of contexts.

Various forms of oral work include:

- listening to adults and children using words correctly;
- acquiring confidence and fluency in speaking, using complete sentences that include the new words and phrases, sometimes in chorus and sometimes individually;
- describing, defining and comparing mathematical properties, positions, methods, patterns, relationships, and rules;
- discussing ways of tackling a problem, collecting data and organising their work;
- hypothesising or making predictions about possible results;

- presenting, explaining and justifying their methods, results, solutions or reasoning, to the whole class or to a group or partner;
- generalising or describing examples that match a general statement.

They need to **read aloud and silently**, sometimes as a whole class and sometimes individually. For example, they should read:

- numbers, signs and symbols, expressions and equations;
- instructions and explanations in textbooks, workbooks, handouts, . . .;
- labels on diagrams, charts, graphs and tables.

They need to **write and record** in a variety of ways, progressing from words, phrases and short sentences to paragraphs and longer pieces of writing. Different forms of writing include:

- writing prose in order to describe, compare, predict, interpret, explain, justify;
- writing formulae, first using words, then symbols;
- sketching and labelling diagrams to clarify their meaning;
- drawing and labelling graphs, charts or tables, and interpreting and making predictions from the data in them, in mathematics and other subjects.

Catering for Individual Needs (Differentiation)

In accordance with Principle 2 of the NMC, *Respect for Diversity* (p. 30), all children should be given the opportunity to achieve to the maximum of their potential.

Children of lower ability need to have the opportunity to experience a range of mathematics, which is appropriate to their level of development, interests and capabilities. Equally children with exceptional ability in mathematics must be extended and not simply be expected to carry out different repetitions of work they have already mastered.
As new experiences cause children to refine their existing knowledge and ideas, so they construct new knowledge. The extent to which teachers are able to facilitate this process significantly affects how well children learn. It is important that they are given opportunities to relate their new learning to knowledge and skills, which they have developed in the past.

Some children fail to reach their potential because they do not see the applicability of mathematics to their daily lives and because they are not encouraged to connect new mathematical concepts and skills to experiences, knowledge and skills they already have. As a result these children develop a negative attitude towards mathematics. The development of more positive attitudes to mathematics and a greater appreciation of its usefulness are the key to improving child participation.

Use of Resources

Manipulatives

The importance of the use of apparatus to help children form mathematical concepts is well known. Using apparatus provides a foundation of practical experience on which children can build abstract ideas. It encourages them to be inventive, helps to develop their confidence and encourages independence.

Teachers need to make use of an appropriate range of apparatus to focus the children's thinking on the concept to be developed, modifying the apparatus as the learner's understanding grows. The use of manipulatives also facilitates the children's thinking during the problem solving process.

Textbooks

Textbooks contain material that provides children with practice and enrichment. They also contain ideas for problem solving situations, which develop mathematical skills and understanding. However teachers must realise that a textbook is just one tool to help with the implementation of the syllabus.

Information and Communication Technology (ICT)

Computers are learning tools which children can use to discover and reinforce new ideas.

ICT can provide children with opportunities to:

- learn from feedback;
- observe patterns;
- see connections;
- work with dynamic images;
- explore data;
- "teach" the computer by giving it simple instructions.

Mathematics Across the Curriculum

Teachers need to help children appreciate the importance of mathematics in their lives. They may achieve this by using the Thematic Approach or by asking colleagues teaching other subjects to provide examples and contexts that may be used in mathematics lessons.

Here are some opportunities that link Mathematics to other subjects:

English

In mathematics general accuracy in using language can be promoted through:

- interpreting questions orally and in writing;
- clarifying the precise meaning of words or mathematical terms;
- discussing the essential ideas identified in the questions and interpreting them to identify the mathematical content;
- creating an awareness of patterns of language by asking children to explain, argue and present their conclusions to others;
- drawing their attention to the statements involved in mathematical reasoning and proof, such as *if ... then*, *because*, *therefore*, …
- stories and poems, such as *Goldilocks and the Three Bears, Five Little Ducks, ...*

Science

Almost every scientific investigation or experiment is likely to require one or more of the mathematical skills of classifying, counting, measuring, calculating, estimating, recording in tables and graphs. Children will, for example, order numbers, including decimals, calculate means and percentages, decide whether it is more appropriate to use a line graph or bar chart and plot, interpret, and predict from graphs.

Art and Craft

Measurements are often needed in Art and Craft. Many patterns and constructions in our own and other cultures are based on properties of shapes, including symmetry and spatial ideas. Designs may need enlarging or reducing, introducing ideas of multiplication or ratio.

History, Geography

Discussing evidence in history or geography may involve measurement, estimation and approximation skills, and making inferences. Children will make statistical enquiries, for example, in analysing population data to explore and compare lifestyles. The study of maps includes the ideas of angle, direction, position and scale.

Physical Education, Music, Drama

Athletic activities use measurement of height, distance and time. Ideas of counting, time, symmetry, pattern (beats and rhythm), movement, position and direction are used extensively in music, dance and competitive games. Role Play provides children with opportunities to relate mathematics to real-life experiences.

Assessment in Mathematics

Evaluation of children's achievement is an essential part of mathematics education. This is necessary for various purposes:

- to give teachers feedback on the success of their methods and approaches and to assist planning for new learning (***formative***);
- to assess the children's readiness for new learning and to find out what they have learnt (***summative***).

Diagnostic assessment procedures enable teachers to discover difficulties that individual children may be having. Appropriate diagnostic assessment may reveal that the reason for a particular student's lack of progress is a lack of understanding achieved at an earlier time and the difficulty may be relatively easily addressed. Diagnosis may also reveal that the child is very talented and is simply bored by lack of stimulation. Diagnostic assessments enable teachers to plan further learning activities specifically designed to meet the learning needs of individual children. Worthwhile diagnosis may be carried out by employing closed and open-type questions.

Assessment should focus both on what children know and can do, and on how they think about mathematics. **It should involve a broad range of tasks and problems and require the application of a number of mathematical ideas**. Skills assessed should include the ability to communicate findings, to present an argument and to exploit an intuitive approach to a problem.

Assessment should be an integral part of the normal teaching and learning programme. It should involve multiple techniques, including written, oral and demonstration formats. Group and team activities should also be assessed.

Teachers should avoid carrying out only tests which focus on a narrow range of skills such as the correct application of standard algorithms (procedures). While such skills are important, a consequence of a narrow assessment procedure, which isolates skills or knowledge, is that children tend to learn in that way. Mathematics becomes for them a set of separate skills and concepts with little obvious connection to other aspects of learning or to their world.

Assessment should also be undertaken to provide children and their parents with an indication of the child's progress. When marking children's work and giving feedback (oral or written) teachers should indicate what the children have done well and what they need to do to improve and to act on feedback given to them. In summarising the results of evaluations of children's achievement, teachers should report what the children have achieved and how well they achieved it. A grade or mark alone is insufficient.

Strands

The Mathematics curriculum is divided into four strands: **Number and Algebra**; **Measures, Shape and Space**; **Data Handling** and **Problem Solving**. This division is a convenient way of categorising the outcomes for mathematics education in schools. It emphasises that there are a number of aspects, which are all equally important. This division does not mean that children are expected to learn Mathematics in individual "packages". On the contrary the **Problem Solving** strand is intended to encourage teachers and children to make connections between the other three strands wherever possible.

Level Descriptors

Each of the four strands in the syllabus (*Number and Algebra, Measures, Shape and Space, Data Handling, Problem Solving*) is divided into eight levels describing the development of the Mathematics curriculum from Year 1 in the Primary School to Form 5 in the Secondary School.

A number of Learning Outcomes are described in each strand and at each level. The outcomes define what children should be able to achieve after appropriate learning experiences in mathematics. They define the progression of learning outcomes, which form the core of the Mathematics curriculum.

At each level the outcomes are quite broad. It is expected that, in assessing children's progress, teachers will make judgements as to an individual's degree of achievement of particular outcomes and will include comments on that degree of achievement when reporting to parents.

It must be recognised that each child is an individual whose learning development and rate of progress is different from others. (Principle 2: *Respect for Diversity* – NMC p.31) Different children will be ready for particular mathematical content and experiences at different times. It is not expected that all children of the same age will be achieving at the same level at the same time, nor that an individual child will necessarily be achieving at the same level in all strands of the mathematics curriculum.

The levels are not meant to be interpreted as the rungs of a ladder, which is to be climbed as quickly as possible. Nor are they meant to be interpreted as hurdles over which each child must pass before moving to any new work. Rather they are meant to focus the mathematics programmes of schools in a consistent way. They provide a basis for describing clearly children's progression in learning.

Lesson Structure

The following structure is recommended for the daily Mathematics lesson:

Year Group: **Years 1 to 6**
Duration: **45 to 60 minutes**

1. **Oral work and mental calculation**
2. **The main teaching activity**
3. **A plenary**

Oral work and mental calculation
(about 5 to 10 min)

- whole class work to rehearse, sharpen, and develop mental and oral skills

The main teaching activity
(about 30 to 40 min)

- teaching input and pupil activities
- work as a whole class, in groups, in pairs or as individuals

It is also recommended that teachers share the specific Learning Outcome/s with the children at the appropriate time during the introductory part of the lesson.

A plenary
(to round off the lesson)
(about 10 to 15 min)

- to sort out misconceptions and identify progress
- to summarise ideas and what to remember
- to set work to do at home
- to make links to other work and discuss the next steps

An additional purpose of the plenary should be to identify progress made vis-à-vis the Learning Outcome. Giving due importance to the Learning Outcome will enhance the children's development of self-assessment skills.

Feedback, in oral and/or written form, should also be related to the Learning Outcome.